Testing for Interactions from Time Series

bivariate surrogate techniques

testing for interactions

given time series of observables from two systems, X and Y

Qs: independent systems? (taking into account eigen-dynamics and type of coupling)

system $\mathbf{X}$	***system*** $\mathbf{Y}$	***coupling***
linear	linear	no coupling
nonlinear	nonlinear	no coupling
linear	linear	linear
linear	nonlinear	linear
nonlinear	linear	linear
nonlinear	nonlinear	linear
linear	linear	nonlinear
linear	nonlinear	nonlinear
nonlinear	linear	nonlinear
nonlinear	nonlinear	nonlinear

testing for interactions

When analyzing bivariate time series from real-world systems

- often can not directly probe for interactions (active experiments)
- many prerequisites of analysis techniques can not strictly be fulfilled
- techniques may be fooled by asymmetries in properties of systems
- need to validate assumptions
- need to interpret values of some measure for strength/direction

$\longrightarrow$ need surrogate methods to test for interactions

testing for interactions — higher-order moments

higher-order moments for some estimators for strength of interaction can be derived to test for nonlinear dependence:

- (linear methods): coherence function κ
- (phase-based methods): mean phase coherence R

disadvantages

consider static nonlinearities only (distinguishable from nonlinear measurement function?)

other approaches:

employ bi-/multivariate surrogates

testing for interactions — higher-order moments

strength of interaction estimated with coherence κ

Q: does a given value of κ indicate nonlinear dependence?

idea: estimate order-*p* spectra

relationships between (poly)spectra, cumulants, and statistical moments

power spectrum $P = FT^{(1)}(AKF^{(2)})$	*p*=2 cumulant $AKF^{(2)}(\tau) \propto x(t)x(\tau)$	*p*=2 statistical moment variance
bispectrum $B = FT^{(2)}(AKF^{(3)})$	*p*=3 cumulant $AKF^{(3)}(\tau) \propto x(t)x(\tau)x(2\tau)$	*p*=3 statistical moment skewness
trispectrum $T = FT^{(3)}(AKF^{(4)})$	*p*=4 cumulant $AKF^{(4)}(\tau) \propto x(t)x(\tau)x(2\tau)x(3\tau)$	*p*=4 statistical moment kurtosis

C.L. Niklas & A.P. Petropulu: Higher-order spectral analysis: a nonlinear signal processing framework. Prentice Hall, New Jersey, 1993
T. Subba Rao & M.M. Gabr: An introduction to bispectral analysis and bilinear time series models. Lecture Notes in Statistics, vol. 24, Springer, Berlin, 1984

testing for interactions higher-order moments

strength of interaction estimated with coherence κ

extension for bivariate case with p=2

<u>def:</u> cross-bispectra and bicoherence (3.order coherence)

$$B^{\mathrm{XXY}}(\omega_{\mathrm{X}},\omega_{\mathrm{Y}}) := \langle \mathcal{F}_{\mathrm{X}}(\omega_{\mathrm{X}})\mathcal{F}_{\mathrm{X}}(\omega_{\mathrm{Y}})\mathcal{F}^*_{\mathrm{Y}}(\omega_{\mathrm{X}}+\omega_{\mathrm{Y}})\rangle_\omega$$

$$B^{\mathrm{XYY}}(\omega_{\mathrm{X}},\omega_{\mathrm{Y}}) := \langle \mathcal{F}_{\mathrm{X}}(\omega_{\mathrm{X}})\mathcal{F}_{\mathrm{Y}}(\omega_{\mathrm{Y}})\mathcal{F}^*_{\mathrm{Y}}(\omega_{\mathrm{X}}+\omega_{\mathrm{Y}})\rangle_\omega$$

$$B^{\mathrm{YYX}}(\omega_{\mathrm{X}},\omega_{\mathrm{Y}}) := \langle \mathcal{F}_{\mathrm{Y}}(\omega_{\mathrm{X}})\mathcal{F}_{\mathrm{Y}}(\omega_{\mathrm{Y}})\mathcal{F}^*_{\mathrm{X}}(\omega_{\mathrm{X}}+\omega_{\mathrm{Y}})\rangle_\omega$$

$$B^{\mathrm{YYY}}(\omega_{\mathrm{X}},\omega_{\mathrm{Y}}) := \langle \mathcal{F}_{\mathrm{Y}}(\omega_{\mathrm{X}})\mathcal{F}_{\mathrm{X}}(\omega_{\mathrm{Y}})\mathcal{F}^*_{\mathrm{X}}(\omega_{\mathrm{X}}+\omega_{\mathrm{Y}})\rangle_\omega$$

$$\mathcal{F}_{\mathrm{X}}(\omega) = FT(x(t))$$

$$\mathcal{F}_{\mathrm{Y}}(\omega) = FT(y(t))$$

$\mathcal{F}^*$ complex conjugate

bicoherence

- is normalized cross-bispectrum (auto-, and cross-spectra)
- is defined on unit interval
- allows detection of quadratic phase-coupling

C.L. Niklas & A.P. Petropulu: Higher-order spectral analysis: a nonlinear signal processing framework. Prentice Hall, New Jersey, 1993
T. Subba Rao & M.M. Gabr: An introduction to bispectral analysis and bilinear time series models. Lecture Notes in Statistics, vol. 24, Springer, Berlin, 1984

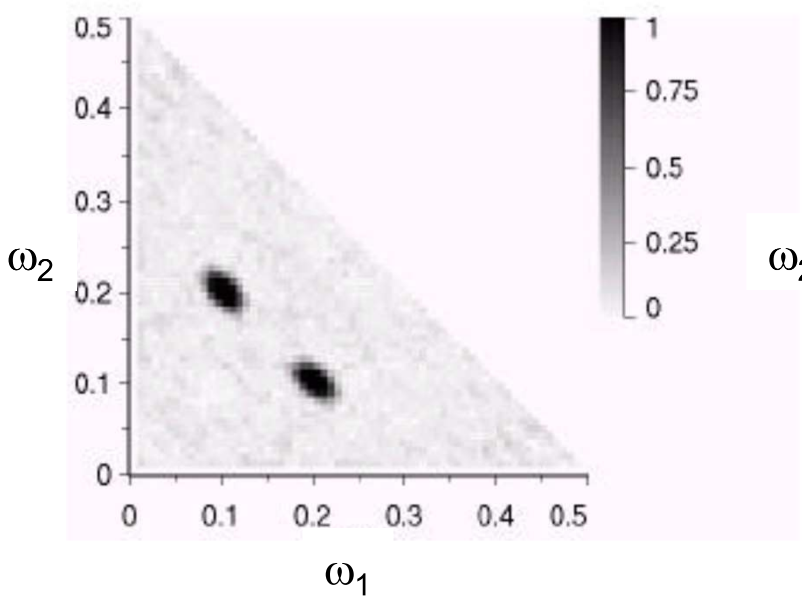
testing for interactions — higher-order moments

strength of interaction estimated with coherence κ

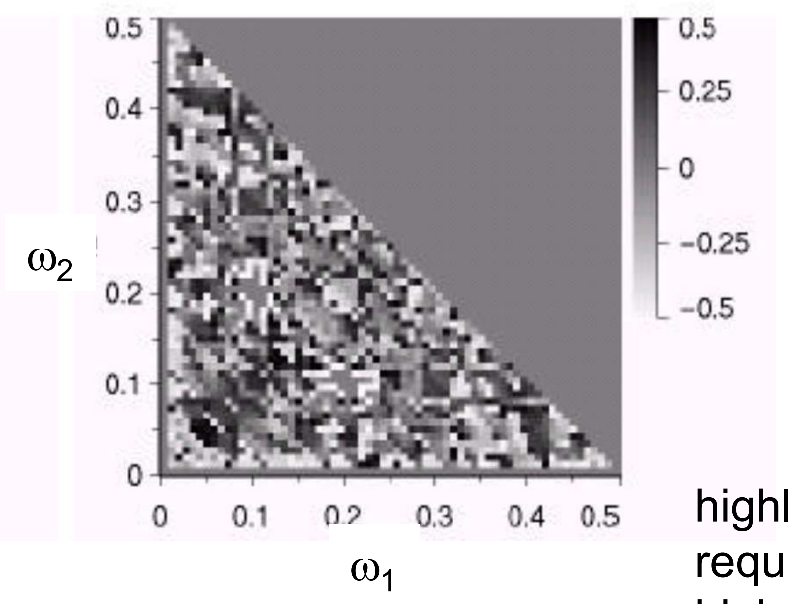
example: 3-wave system with quadratic phase coupling

$$y(t) = \cos\left(2\pi(\omega_1 t + \phi_1)\right) + \cos\left(2\pi(\omega_2 t + \phi_2)\right) + \cos\left(2\pi(\{\omega_1 + \omega_2\}\, t + \phi_1 + \phi_2)\right)$$

modulus of bicoherence



phase of bicoherence



highly sensitive to noise
requires large number of data points
high computational cost

testing for interactions higher-order moments

strength of interaction estimated with mean phase coherence R

Q: does a given value of R indicate nonlinear dependence?

idea: estimate higher-order moments of phase-difference distribution

general ansatz:

given $\Delta\phi_{\mathrm{XY}}(t) := n\phi_{\mathrm{X}}(t) - m\phi_{\mathrm{Y}}(t)$

define order-p moments of phase-difference distribution on unit circle

$$m_p := a_p + ib_p$$

where

$a_p := \frac{1}{N}\sum_{n=1}^{n}\cos(\Delta\phi_{\mathrm{XY}}(n));\quad b_p := \frac{1}{N}\sum_{n=1}^{n}\sin(\Delta\phi_{\mathrm{XY}}(n))$

$$\rightarrow m_p := R_p \mathrm{e}^{im_p}$$

KV Mardia, P Jupp. *Directional Statistics*, John Wiley and Sons Ltd., 2000

testing for interactions higher-order moments

strength of interaction estimated with mean phase coherence R

$$\text{mean of distribution:} \quad m_1 := R \quad (= R_1)$$

$$\text{variance of distribution:} \quad m_2 := S = 1 - R$$

$$\text{skewness of distribution:} \quad m_3 := \frac{R_2 \sin(m_2 - 2m_1)}{m_2^{3/2}}$$

$$\text{kurtosis of distribution:} \quad m_4 := \frac{R_2 \cos(m_2 - 2m_1) - (1 - m_2)^4}{m_2^2}$$

$m_3 = m_4 = 0$ for unimodal symmetric distribution

deviations might indicate nonlinear dependence

KV Mardia, P Jupp. *Directional Statistics*, John Wiley and Sons Ltd., 2000

testing for interactions **surrogates**

bi-(multi-)variate surrogates

first ideas: extend concepts of (iteratively amplitude-adjusted) phase-randomized surrogates (FT, IAAFT → MFT, MIAAFT)

given: simultaneously recorded time series, with zero mean and unit variance

assumption: dynamical nonlinearity associated with changes in phase difference

requirements: MFT surrogate must reproduce linear correlations within and between time series (need to estimate auto- and cross-spectrum)
MIAAFT must, in addition, reproduce the respective amplitude distributions

Prichard & Theiler, PRL 73, 951, 1994; Schreiber & Schmitz, Physica D 142, 346, 2000

testing for interactions **surrogates**

bi-(multi-)variate surrogates

MFT null hypothesis:
the data have been generated by linearly correlated, stochastic, stationary processes with Gaussian distributed amplitudes, and possibly observed through some (static) nonlinear measurement function

method:
- Fourier transform to preserve linear properties
    cross-spectrum (Wiener-Khinchin theorem: cross-correlation)
- randomize nonlinear properties
    replace phase-differences with random numbers $[0, 2\pi)$
- inverse transform
- caveat: does “correlated” imply “coupling”?

testing for interactions surrogates

bi-(multi-)variate surrogates

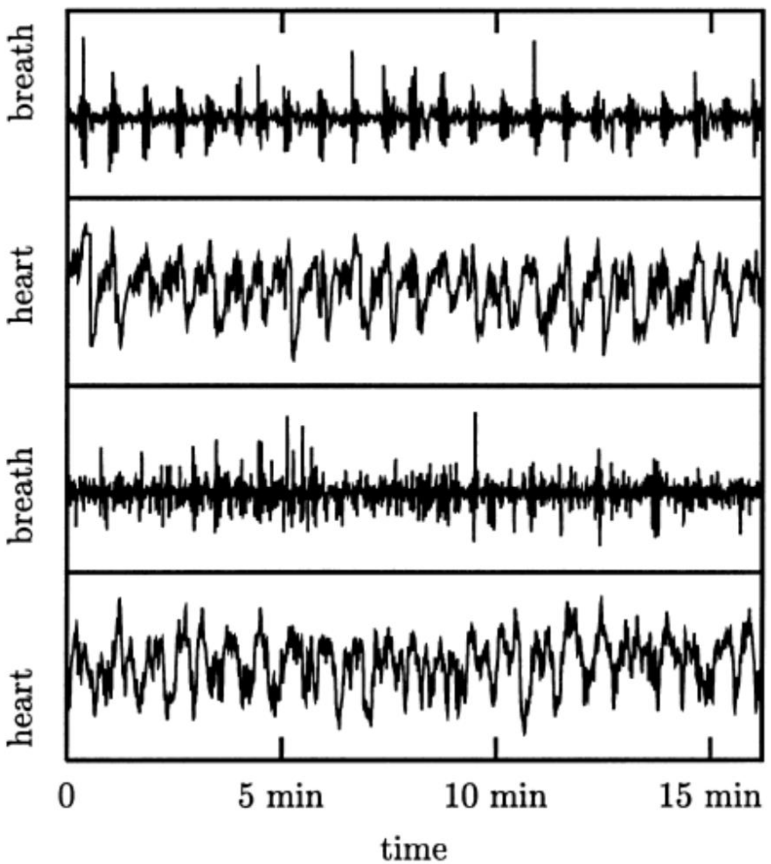
MIAAFT null hypothesis:
the data have been generated by linearly correlated, stochastic, stationary processes with arbitrary amplitude distributions, and possibly observed through some (static) nonlinear measurement function

method (cf. IAAFT):
in addition, replace phase-differences with random numbers $[0, 2\pi)$
but preserve original phase-difference properties as best as possible
- slightly modify rank-sequence of phase-difference (additive perturbation)
- replacement of phase-differences in least-squares sense
- caveat: does “correlated” imply “coupling”?

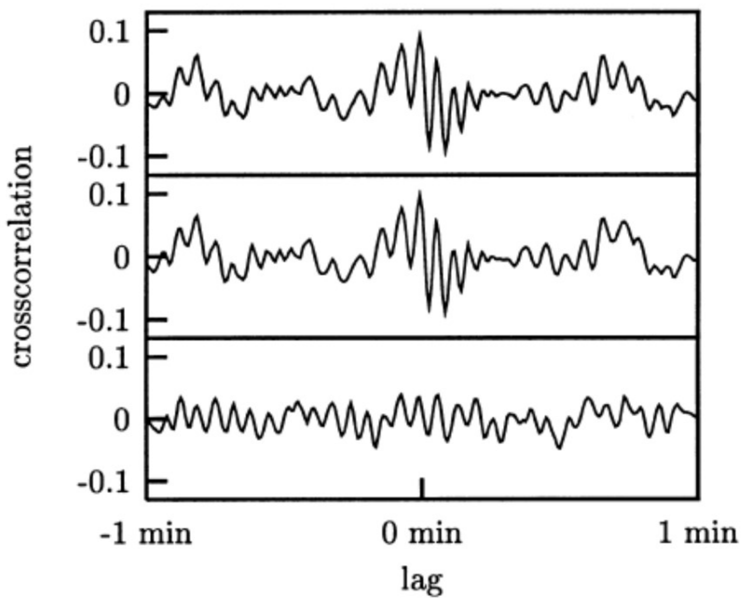
testing for interactions surrogates

MIAAFT surrogates examples

T. Schreiber, A. Schmitz/Physica D 142 (2000) 346–382



Simultaneous surrogates for a bi-variate time series. The upper two panels show simultaneous recordings of the breath rate and the instantaneous heart rate of a human. The lower two panels show surrogate sequences that preserve the individual distributions and power spectra as well as the cross-correlation function between heart and breath rate. The most prominent difference between data and surrogates is the lack of coherence in the surrogate breath rate.



Cross-correlation functions for the bi-variate data shown in left Fig. (upper panel), and a surrogate that preserves the individual spectra and distributions as well as the relative Fourier phases (middle). The lower panel shows the same for surrogates prepared for each channel individually, i.e. without explicitly preserving the cross-correlation structure.

testing for interactions surrogates

bi-(multi-)variate surrogates

extension for near-coherent data:
coherent digitally filtered surrogate (CDF)

assumption: dynamical nonlinearity associated with properties of coherence function

requirements: CDF surrogate must preserve spectrum, cross-spectrum, and coherence function

method:
-generate frequency-dependent alternative coherence function and its inverse (→filter function)
-multiply spectra with filter function

Dolan & Neimann, PRE 65, 026108, 2002

testing for interactions surrogates

time-shifted (TS) time series as surrogates

TS null hypothesis:
the data have been generated by uncoupled processes with arbitrary structure, and possibly observed through some (static) nonlinear measurement function

method:
- quasi-continuous time shifting or choose set of random delays
- periodic boundary condition (wrap around at end of shifted time series to its beginning) preserves total length of time series
- state-space trajectory invariant under shifting operation for periodic boundary condition
- caveat: distinguish linearity from nonlinearity?

R. Quian Quiroga, A. Kraskov, T. Kreuz, P. Grassberger, Phys. Rev. E 65, 041903, 2002; T.I. Netoff, S.J. Schiff, J. Neurosci. 22, 7297, 2002

testing for interactions surrogates

bi-(multi-)variate surrogates from constrained randomizations (CR)

CR null hypothesis:
the data have been generated by linearly coupled processes with arbitrary structure, and possibly observed through some (static) nonlinear measurement function

method:
- would need to maintain time series and cross correlation …
  what to randomize?
- idea: preserve auto- and cross-correlation up to certain maximum delay (iteratively minimize appropriate cost function, simulated annealing
- caveat: high computational cost; convergence?

T. Schreiber, Phys. Rev. Lett. 80, 2105, 1998; RG Andrzejak, A Kraskov, H Stögbauer, F Mormann, T Kreuz, Phys. Rev E, 68, 066202, 2003.

testing for interactions **surrogates**

special bi-(multi-)variate surrogates for nonlinear interdependencies

null hypothesis:
the data have been generated by linearly coupled processes with arbitrary structure, and possibly observed through some (static) nonlinear measurement function

method:
- for conditional measures ($S(\mathrm{X}|\mathrm{Y})$), randomize time series of system X (e.g. IAAFT), and vice versa, before delay-embedding
- use "randomized" time indices for distance calculations

-caveat: may be not useful for estimating strength of interaction

RG Andrzejak, A Kraskov, H Stögbauer, F Mormann, T Kreuz, Phys. Rev E, 68, 066202, 2003.

testing for interactions **surrogates**

special bi-(multi-)variate surrogates for (symbolic) transfer entropy

null hypothesis:
the data have been generated by uncoupled processes with arbitrary structure, and possibly observed through some (static) nonlinear measurement function

method:
- randomize time series of (presumed) driving system either in time domain (e.g. IAAFT) or in state-space (e.g. random shuffling of state-space vectors)

R. Marschinski, H. Kantz, Eur. Phys. J. B **30**, 275, 2002; A. Papana, D. Kugiumtzis D, PG Larsson. Phys. Rev. E. 83, 036207, 2011

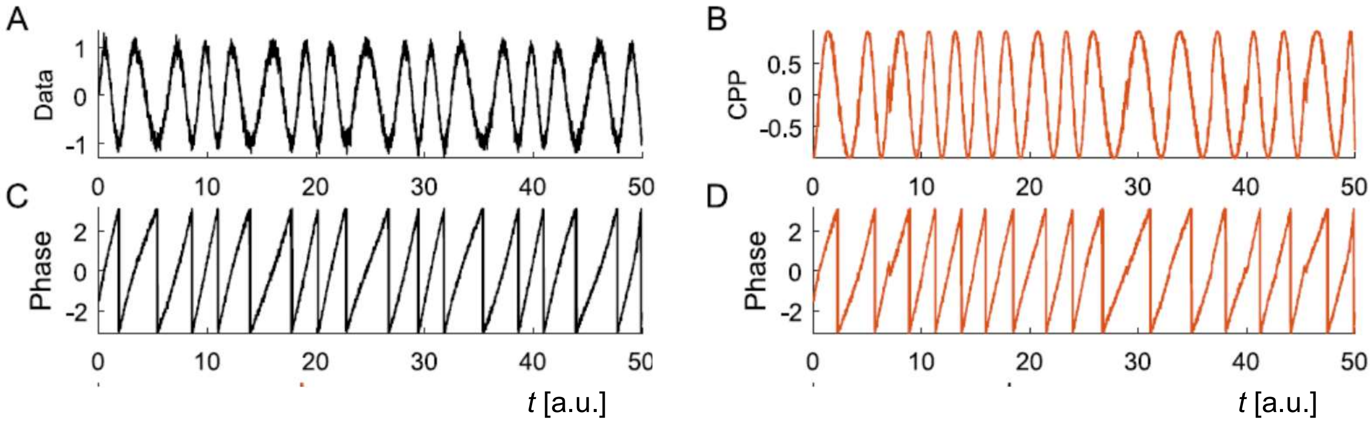
testing for interactions **surrogates**

cyclic phase permutation (CPP) surrogates

null hypothesis:
the systems have independent phase dynamics

method:
- derive phase time series (e.g. Hilbert transform)
- find individual cycles (periodicities) and randomly permute phase cycles

example:
noisy sine-wave with
time-varying periodicity



- caveat: null hypothesis sufficient?

Lancaster G, Iatsenko D, Pidde A, Ticcinelli V, Stefanovska A. Phys. Rep. 748, 1-60, 2018

testing for interactions

rejecting/accepting null hypothesis

choose significance level

$p = (1-\alpha) \times 100\%$

$N_s = (1/\alpha - 1)$ surrogates (one-sided)
$N_s = (2/\alpha - 1)$ surrogates (two-sided) ← original time series
(e.g., $N_s = 19$ resp. 39 for $p = 95\ \%$

measure for interaction differs for originals and surrogates
↓
reject null hypothesis: indication for dependence

measure for interaction does not differs for originals and surrogates
↓
accept null hypothesis: no indication for dependence

testing for interactions

rejecting/accepting null hypothesis

interpretation:

- statistical test only, it is not a proof for dependence!
- rejection of null hypothesis only provides necessary but not sufficient condition for interaction measure to indicate dependence
- acceptance of null hypothesis does not indicate its correctness
- whenever a null hypothesis is rejected, it is always very important to keep in mind that the complementary hypothesis is very comprehensive and might include many different reasons that are possibly responsible for this rejection
- consider including other (statistical) properties of your time series and properties of the coupling into the null hypothesis

testing for interactions surrogates

state-of-the-art:

Fourier phase randomization alone may be inappropriate, because of mixing of phase and amplitude information

specifically designed surrogates require a priori knowledge about systems and data

no best method, no unified concept

unsolved issue:
how to discern a linear superposition of independent nonlinear deterministic dynamics from coupled nonlinear deterministic dynamics?

testing for interactions

bias correction for measures of direction of interaction

measures for direction of interaction should be zero in case of absent directional couplings

non-zero values may be due to various reasons
- statistical issues (e.g. insufficient number of data points)
- sensitivity of measure
- “similar” systems
- directional coupling

idea: use surrogate to correct for potential bias

testing for interactions

bias correction for measures of direction of interaction

ansatz:

let $D_{\mathrm{X}\to\mathrm{Y}}$ denote a measure for directionality between systems X and Y

define bias-corrected measure $D^c_{\mathrm{X}\to\mathrm{Y}}$ as

$$D^c_{\mathrm{X}\to\mathrm{Y}} = D_{\mathrm{X}\to\mathrm{Y}} - \frac{1}{N_{\mathrm{surr}}} \sum_{i=1}^{N_{\mathrm{surr}}} D^{(i)}_{\mathrm{X}\to\mathrm{Y}}$$

derive surrogates from randomization of time series of system X (using methods discussed before)

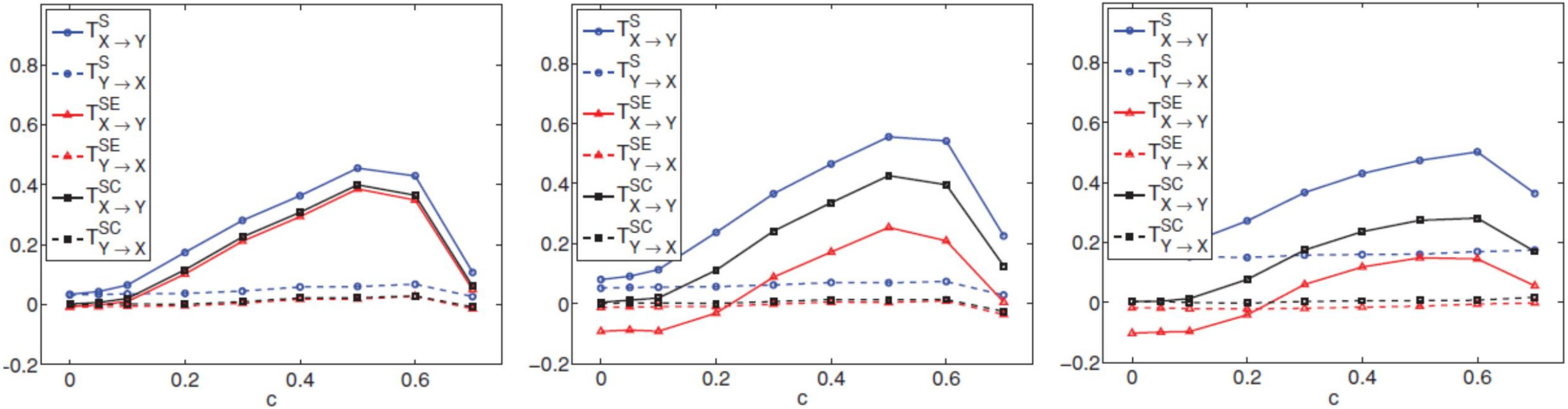
$D^c_{\mathrm{Y}\to\mathrm{X}}$ is defined in complete analogy

testing for interactions

bias correction for measures of direction of interaction

example:
unidirectionally coupled Hénon maps, 512 data points each
symbolic transfer entropy, different correction schemes, mean from 100 realizations
different embedding dimensions, noisy-free case, + 20 % noise

$m_x = m_y = 3$ $m_x = 4; m_y = 3$ noisy systems



Papana A, Kugiumtzis D, Larsson PG. Reducing the bias of causality measures. Phys. Rev. E. 83, 036207, 2011

testing for interactions

bias correction for measures of direction of interaction

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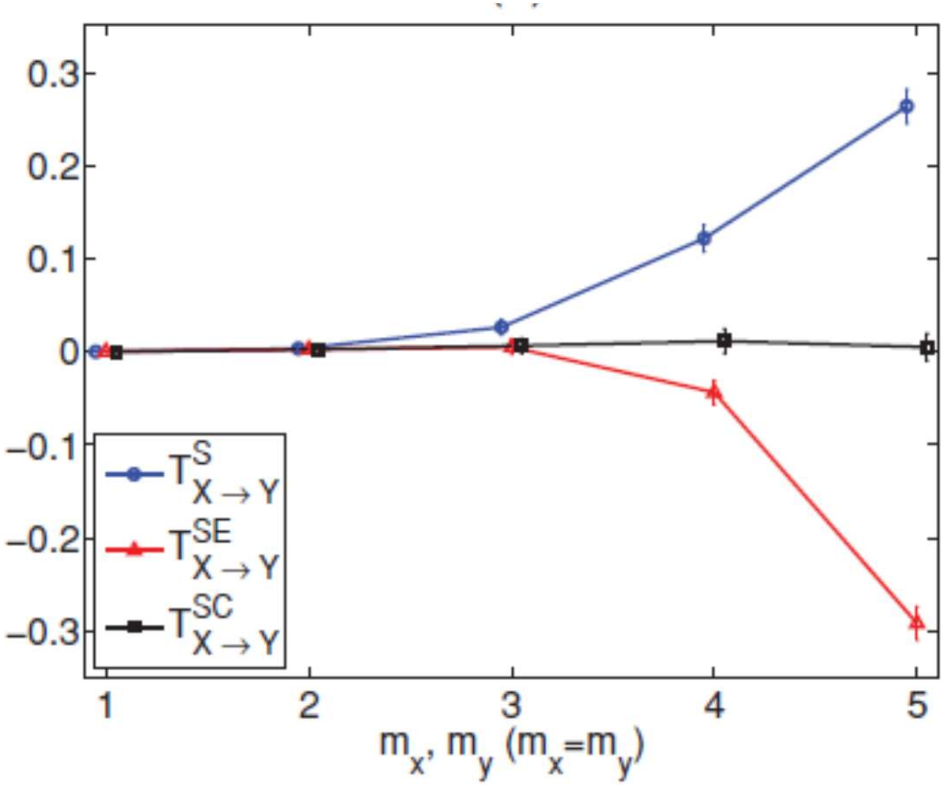

Papana A, Kugiumtzis D, Larsson PG. Reducing the bias of causality measures. Phys. Rev. E. 83, 036207, 2011

testing for interactions what can go wrong?

field applications

- all issues related to estimating a given interaction measure
- all issues related to univariate surrogates
- all issues related to formulating adequate null hypotheses

- avoid wishful thinking!
  (sometimes it’s just P2C2E*)

*process too complicated to explain: Iff in S. Rushdie: Haroun and the Sea of Stories. Granta Books, London (1990)